

## Advanced Combined Bundle Block Adjustment with Kinematic GPS Data

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**ABSTRACT:** This paper introduces a newly developed, mathematical rigorous method which enables GPS data recorded during a photo flight to be introduced directly into the bundle adjustment without the block being weakened with additional shift and drift parameters. For the first time in an operational software package for bundle adjustment, all the advantages of the GPS data can be used to the full without having to fly additional cross strips at the edges of the block.

**RÉSUMÉ:** Dans l'article suivant une méthode nouvelle et mathématiquement exacte est présentée qui permet d'introduire des données GPS collectionnées pendant un vol de photographies aériennes directement dans la compensation des faisceaux photogrammétriques, sans amoindrir le bloc par des paramètres supplémentaires d'équilibre et de direction. Ainsi pour la première fois on peut utiliser entièrement tous les avantages des données GPS dans un programme opérationnel pour le traitement numérique des faisceaux photogrammétriques sans être obligé de survoler des bandes transversales en plus aux bords des blocs.

**KURZFASSUNG:** Es wird eine neu entwickelte, mathematisch strenge Methode vorgestellt, die es gestattet, GPS Daten, die während eines Bildfluges aufgezeichnet wurden, direkt in die Bündelausgleichung einzuführen, ohne den Block durch zusätzliche Shift- und Driftparameter aufzuweichen. Damit können erstmals in einem operationellen Softwarepaket zur Bündelausgleichung alle Vorteile der GPS Daten voll genutzt werden, ohne zusätzliche Querstreifen an den Blockrändern fliegen zu müssen.

### 1. Background

Continual progress in GPS technology as well as in receiver hardware and analysis software also leads to increases in accuracy and cost saving in photogrammetry. Improved control of survey aircraft and aerial cameras contributes towards reliability during the flight. The start of aerial strips as well as end and side laps can be kept to more accurately. Overlapping with neighboring strips can also be controlled accurately for each photograph. Conjugated photographs from neighboring strips overlap completely lengthwise. It is considerably easier to select tie points and the number of these points is reduced.

For the subsequent bundle block adjustment, the GPS and camera data recorded with relation to the antennae positions or the projection centers is processed for all exposure times. In the process a few imponderables ensue: The GPS position is determined in a fixed rhythm of e.g. half a second. In this half second, the aerial survey craft covers a distance of approx. 35

m. An interpolation over this distance leads to some losses in accuracy.

During the GPS data analysis, due to the geometry of the satellite configuration and the wave lengths of the signals, phase ambiguities have to be resolved in order to obtain correct results. For ground stations this is done using a longer observation time. In the aircraft, dependent on the number of received satellites, the flight strip length, the stability of the reception in the individual strips, the inclination of the aircraft in the turning loops, which could lead to loss of signals, and the constancy of the weather conditions, it may be difficult or impossible to resolve these phase ambiguities. In individual cases this may result in incorrect solutions. Such uncertainties and errors cause a misalignment of the exposure positions and in addition a falsification, which is essentially time-dependent, of the coordinates. The altitude components are particularly sensitive to those errors.

As a result of the GPS analysis, the exposure positions are obtained in geocentric coordinates in the WGS84 datum. These are then converted to the national coordinate system. However, there is not always sufficient information for the datum transformation.

In the subsequent bundle block adjustment, these positions are considered as observations. In other solution approaches which exist to date, in order to recognize and rectify possible coordinate falsifications, shift and drift parameters are introduced into the block adjustment as additional unknowns. 6 parameters are obtained for each flight section if the satellite configuration remains the same and the reception is constant. The position function is replaced by the time. This is permitted in the first approximation if the aircraft flies on a straight course. Also the non-linearity of the influences of the ambiguities can be modeled without any disadvantages by a linear drift formulation for flight strips which are not too long. The number of additional unknowns which 'weaken' the block can quickly become very high.

The conditions required for the shift and drift formulation of the unchanging satellite constellation and the uninterrupted signal reception partly lead in practice to considerable information loss. This is because satellites which have only been observed during a part of a flight section, or for which a cycle slip has occurred, have to be removed from the analysis. Due to this a weakening of the geometry results which in turn causes a greater non-linearity of the influence of the ambiguities as well as lower accuracies.

To ensure the determinability of shift and drift parameters, it is thus necessary to increase the number of control points or to introduce cross strips for block stabilisation. The possible gain from including GPS data is at least partly lost again.

The shift and drift parameters model, in addition to the non resolved phase ambiguities, simultaneously time errors from uncorrected datum transformations and non modelled influences of a changing troposphere. Even if individual shift and drift parameters are not significant and thus could actually be taken out of the adjustment, they often have to be retained due to the aforementioned reasons as they are physically justified.

A further problem is the correct weighting of the GPS data in the bundle block adjustment. Due to incorrect weighting formulations, accuracies and reliabilities can easily be reproduced which have nothing to do with the reality.

## 2. A new approach

A combination of the known software packages GEONAP and BINGO-F considerably improves the functional approach with the help of a rigorous mathematical model. After the GPS data has been preprocessed by GEONAP, the unknown parameters of the phase ambiguities are edited as design matrix for BINGO. Thus in the block adjustment all phase

ambiguities are resolved directly taking into account the photogrammetric image measurements and the photogrammetric control points. With a purely GPS data analysis, this additional information is not available.

For this step it is necessary to transform the GPS coordinates and their partial design matrix to the national coordinate system. GEONAP executes these tasks.

This design matrix is then read by BINGO-F and integrated smoothly into the equation system of the photogrammetric block adjustment. For the initial constellation of the satellites, at most one new unknown comes into the equation system per satellite. In addition at most only one new unknown is added for each change of the satellite configuration and for each cycle slip (in contrast to up to 6 new unknowns per strip using conventional methods). Thus the block becomes very much more stable. Additional points for stabilising the block are no longer required. By the time this paper is introduced at the congress, it will be possible to see, based on empirical data, whether - as assumed - additional cross strips on the block edges can also be dispensed with.

As naturally even with these very rigorous methods, the datum transformation - not always known to begin with - has to be modelled from WGS84 to the national system, 7 transformation parameters are also carried along as additional unknowns which transfer the GPS measurement data to the national system. In this way a rigorous separation is achieved between the parameters introduced due to various physical reasons.

Possible influences of an atmosphere varying from location to location are to a greater extent recorded by the parameter formulation. Alternatively, it is possible to use an additional parameter to describe this influence as well.

For the first time, using BINGO-F, the weighting problem is also solved satisfactorily. The weighting formulations are checked group by group using the variance component estimation and thus estimated directly on line in each bundle adjustment. There is no need to rely on extensive and expensive empirical investigations for accuracy.

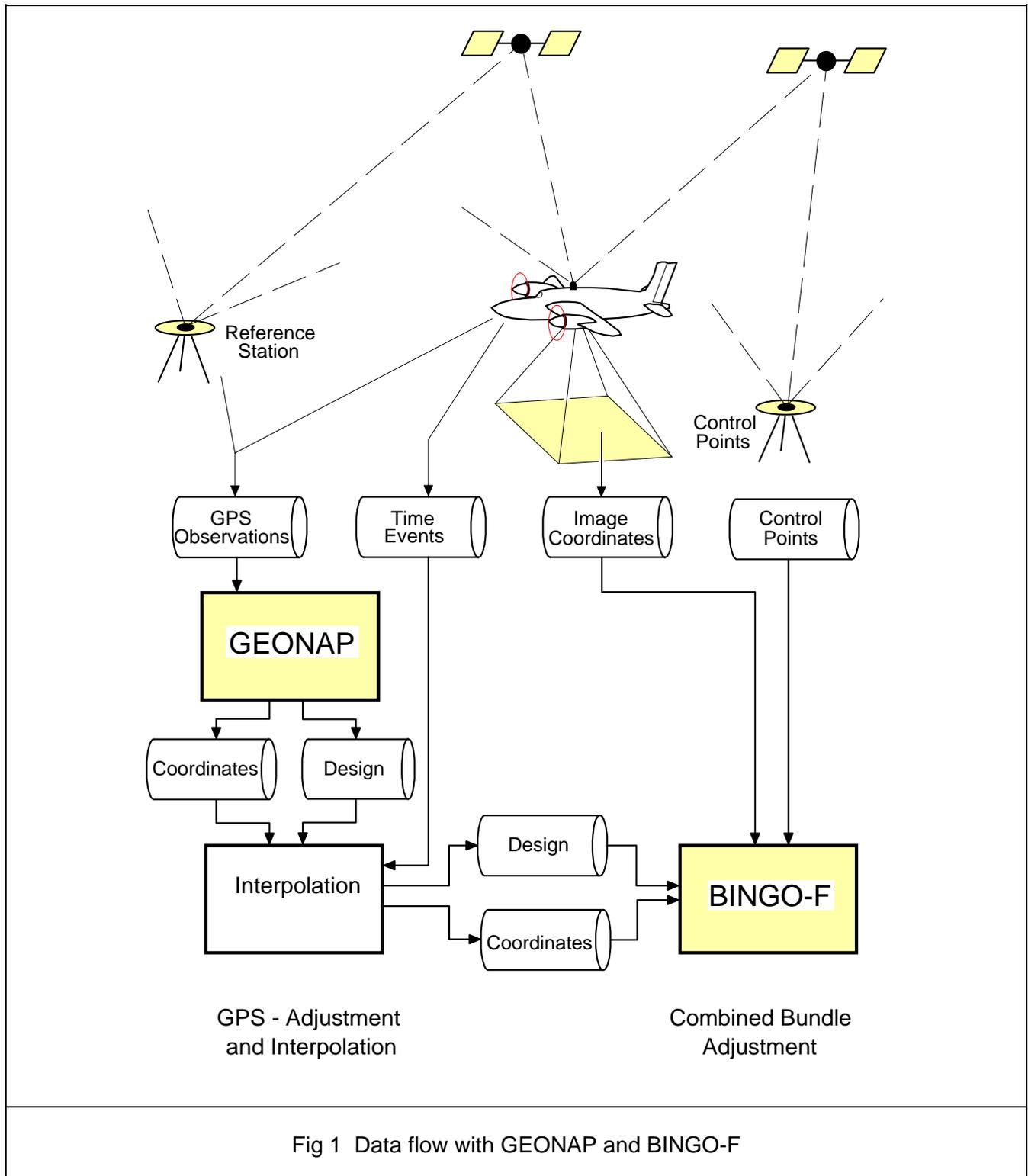
## 3. GEONAP

GEONAP is a powerful multi-purpose GPS adjustment program for static and kinematic GPS observation. It allows a strict evaluation of multiple stations with undifferenced carrier phase observations which is important if more than one GPS reference station is used during the flight.

GEONAP has implemented all modern techniques for the resolution of the carrier phase ambiguities and therefore allows reliable results with high precision. However, under some circumstances not all ambiguities may be resolved. The remaining unresolved ambiguities are normally left in the observation equations as non integer values. For photo flights, where a high

relative accuracy within strips is required, the remaining ambiguities are fixed to the nearest integers, regardless of statistical indicators. Information on

unresolved (and forced fixed) ambiguities is provided by GEONAP to the subsequent bundle block adjustment program BINGO-F.



#### 4. BINGO-F

BINGO-F is a modern operational combined block adjustment program for aerial and terrestrial photogrammetric application. It enables the inclusion of photo coordinates and control point coordinates as well as all types of survey measurements, GPS data

and several types of object conditions. Gross errors can be eliminated automatically according to the rigorous method of Baarda. To differentiate between data and weighting errors, the variance component estimation is included.

#### 5. Data Processing

For each position-coordinate estimated in GEONAP, the corresponding design matrix elements for the unresolved ambiguities are computed. The design matrix is mainly a function of the satellites' current elevation and azimuth relative to the aircraft's position.

GEONAP recognizes the times where cycle slips have occurred. It can assign a counter for the current ambiguity for each epoch. If the ambiguity of a satellite was successfully fixed, it need not be considered for the subsequent BINGO-F adjustment for this epoch. For an unresolved ambiguity of a satellite, however, an additional unknown has to be introduced into the BINGO-F adjustment. This unknown stands in the BINGO-F adjustment until a new (resolved or unresolved) ambiguity occurs for this satellite.

This yields to one additional unknown to be estimated in BINGO-F for every ambiguity that was left unresolved from GEONAP. In the worst case GEONAP cannot solve any ambiguity. In this case the maximum number of unknowns is one unknown per satellite and one additional unknown for each cycle slip occurred. In the best case, when GEONAP has solved all ambiguities, no additional unknown has to be carried along in BINGO.

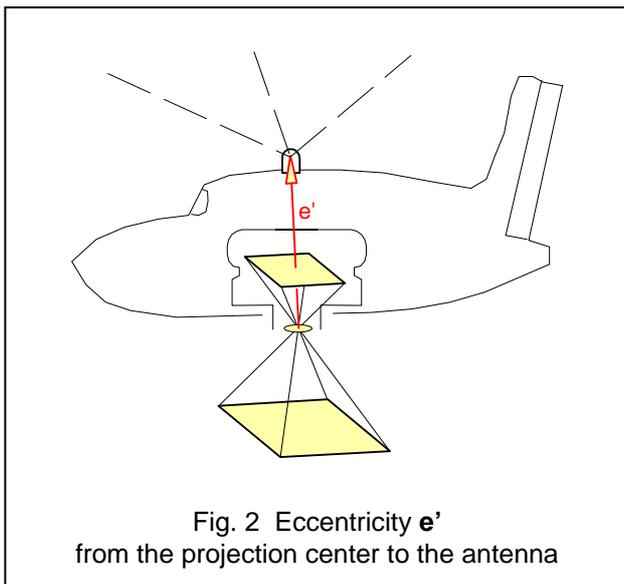


Fig. 2 Eccentricity  $e'$  from the projection center to the antenna

The last step after the computation of the aircraft's GPS antenna position is the interpolation of the camera exposure events between the GPS exposure events. The reduction from the GPS antenna position to the camera's projection center is also done in this step. BINGO-F enables the estimation of the vector  $e'$  from the projection center to the antenna (see Fig. 2) as additional unknown in the adjustment process.

In most cases it suffices to derive the current orientation of the aircraft platform from the GPS trajec-

tory. Sometimes it may be useful to take additional information into account for this purpose, e.g. from aircraft sensors. The interpolation should consider changes in the satellite configuration and unresolved cycle slips during the interpolation interval.

## 6. Example

Let us assume a configuration of 7 satellites, a smooth flight with no cycle slips, and a flight of 4 strips. Let us assume additionally that all the ambiguities are left unresolved. With the usual approach there are 6 additional unknowns to be estimated per strip: 3 shift and 3 drift parameters. This yields to 24 additional unknowns for the flight. With the new approach, only 6 additional ambiguity parameters for the 7 satellites (one is linear dependent on the 6 others) have to be estimated for the whole flight.

## 7. Conclusions

Advantages of the new approach:

- only a minimum of required parameters have to be estimated.
- no information in the GPS adjustment is lost.
- the mathematical model is functionally correct, no problems with possible non linear configurations.
- the ambiguity resolution is possible in iterative steps, first in GEONAP and second in BINGO-F, and it is possible to use the additional information from the photo measurements for the sophisticated ambiguity search algorithms.

It can already be said today that this method will gain acceptance due to economical reasons as costs can be saved for each photogrammetric block adjustment leading to a short pay-back period for the software development.

## 8. Literature

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