

Seminar on Photogrammetric Mapping from SPOT Imagery
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B I N G O

Bundle Block Adjustment Program

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Bundle Block Adjustment Program for Close Range Photogrammetry, aerial triangulation, SPOT Data, and three-dimensional geodetic network adjustment

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1. Introduction

The BINGO adjustment software package has been designed for practical use in many areas of photogrammetry. Special emphasis has been placed not only on a precise mathematical model but also on flexibility and user-friendliness. BINGO can be used for terrestrial and aerial photogrammetry, for SPOT applications and for three-dimensional geodetic network adjustment.

A program for terrestrial photogrammetry was presented in 1978 already. This program can be said to be BINGO's precursor. The first small BINGO version with limited capabilities was presented in 1981. The following development phases are briefly described below (Table 1).

Today BINGO is being used all over the world on different computers (Table 2). Installation on further computers presents no problems.

1981	First small BINGO version for terrestrial photogrammetry; limited capabilities.
1982	Expansion for large blocks, camera calibration, and additional parameters.
1983	Survey measurements, aerial triangulation, data snooping.
1984	Variance component estimation, Fortran 77, VAX-11 version, Nova version.
1985	German and English manuals, more than 200 pages each.
1986	HP-1000 version, linkage with the C-Series <u>Planicom</u> SPOT version (first trials since 1984)
1987	Automatic computation of approximate photo orientation. Linkage with P-Series <u>Planicom</u> (being prepared).

Table 1 BINGO Development Phases

Available for	HP-1000 A Series, <u>Planicom</u> , <u>Orthocom</u> VAX 11 PRIME NOVA / DATA GENERAL SUN SPERRY / UNIVAC IBM AT, MS-DOS (nearly completed) GOP - CONTEXT VISION (nearly completed)
Also used in the past on	CDC - CYBER 76 CRAY 1

Table 2 BINGO Installations

2. Fields of Application

The BINGO software package has been developed for the combined adjustment of photogrammetric bundles, geodetic networks and further observations and conditions. It can be used for the following tasks:

- *Bundle block adjustment*
 - . in engineering applications
 - . in aerial triangulation
 - . of space photographs
 - . of SPOT satellite images

- *Calibration of metric cameras*
 - . with test fields
 - . simultaneously with bundle block adjustment

for determining the camera constant, the principal point position and the image deformation. The image deformation is determined by means of up to 24 additional parameters according to Jacobsen /1982/. Required and useful parameters are selected automatically.

- *Geodetic network adjustment*

The package includes three-dimensional geodetic network adjustment. The following observations can be used: sets of directions, azimuths, zenith distances, three-dimensional distances, horizontal distances, coordinate and elevation differences, vertical lines, local levelling networks and, with certain narrow restrictions, straight-line and plane conditions.

- *Combined adjustment*

From a statistical point of view the combined adjustment can always be expected to yield the best results. Photographs taken with any cameras either on the ground or from the air can be combined. It is possible to calibrate selected cameras simultaneously. The mentioned survey measurements can all be included in the adjustment process.

Photogrammetric observations and conditions such as, for example, stereometric camera conditions can also be included if stereometric cameras are used, as well as terrestrial photogrammetry camera station conditions.

The orbit data can be used as measurement information for SPOT image adjustment.

- *Simulation computation*

Networks can be simulated easily without physical observation material. Such computations are useful for determining the optimum design of a network. Simulation can also be used for immediate precision and reliability assessment after data acquisition. A simulated Sigma Naught can be used to determine the final precision and reliability without having to locate existing data errors that degrade the precision information.

- *Adjustment of defective networks*

Networks become rank-defective if the 7 datum parameters are not defined precisely (datum defect) or if some points are not linked appropriately in the network (configuration defect). BINGO overcomes defects by transforming non-determinable unknowns to their approximations. The defects are analyzed simultaneously to provide precise information to the user.

As a result, networks can be adjusted even before all of the measurement data has become available.

3. Mathematical Model

3.1 Functional Model

The redundant system is adjusted by means of the least squares method.

The functional model is described by the matrix equation

$$v = Ax - l \quad (2)$$

where A is the design matrix and represents the relationship between the unknowns and the observations.

The following observations can be used:

- *Image coordinates*
of terrestrial and aerial photographs and SPOT images. Different rotation angles can be used for terrestrial and aerial photographs.
- *Orientation data*
Known camera stations, for example, can be used as observations for adjusting terrestrial photographs, or the fixed relative orientation data of photo pairs in the case of stereometric camera photographs, or the orbit data in the case of SPOT images.
- *Survey measurements*
In addition to control points, all of the above angle and distance data can be used with the functional model.

The equation $v = Ax - l$ establishes a linear relationship between the unknowns x and the observations l . The non-linear original equations are developed into Taylor series for linearization. This involves the problem of computing approximations for all unknowns, which will be discussed later (chapter 4).

3.2 Stochastic Model

Each observation l is assigned a standard deviation q_{ll} which describes the stochastic behavior, and this data is used to establish the matrix P_{ll} . Since non-correlated observations are assumed, P_{ll} is a diagonal matrix. BINGO affords very flexible weighting.

The stochastic behavior of the adjusted unknowns is given by:

$$s_0^2 \cdot Q_{xx} = s_0^2 \cdot (A^T P A)^{-1} \quad (2)$$

The following applies to the adjusted residuals:

$$Q_{vv} = Q_{ll} - A Q_{xx} A^T \quad (3)$$

The Q_{vv} matrix is very complicated to compute but is of major significance: It serves to assess the inner reliability and to locate gross errors.

BINGO partially computes the Q_{xx} matrix to provide standard deviations for all adjusted unknowns. Of Q_{vv} only the diagonal is computed because only these elements are required for further computations.

In contrast to other adjustment programs which approximate Q_{xx} and Q_{vv} only roughly, real computations are performed here. This has been made possible by new computation and storage methods and a new optimization method that have been described in the literature /Kruck 1983, 1984, 1985/.

The diagonal elements of the geometry matrix $Q_{vv} \cdot P_{ll}$ are called redundancy numbers.

$$r_i = (Q_{vv} \cdot P_{ll})_{ii} \quad (4)$$

The r_i values indicate the inner reliability.

3.3 Data Snooping

The redundancy number r_i indicates the component of an existing error of an observation l_i that will appear in the associated residual v_i . This can be used to compute the normalized residual according to Baarda:

$$w_i = \frac{v_i}{s_{vi}} = \frac{v_i}{s_o} \cdot \sqrt{\frac{p_i}{r_i}} \quad (5)$$

$|w_i|$ is tested against a bound k . This test is used very successfully in BINGO for detecting data errors.

3.4 Variance Component Estimation

The result of an adjustment can vary widely depending on the weight used /Kruck 1985/. Especially if quite different observation data is used, which is easily possible in BINGO, careful weight monitoring is therefore a must.

An algorithm proposed by Förstner /1979/ is used for estimating the group weights, and for providing accuracy data on the chosen weights to the user in observation group order.

4. Orientation Data

In the case of SPOT images the orientation parameters are given, and in the case of terrestrial photography they can in part be measured. In the case of aerial triangulation, approximate orientations are easy to compute because known facts facilitate the computations considerably (e. g. approximately vertical photography, strip layout).

In BINGO, however, the problem is that approximate orientations have to be computed for completely random taking arrangements, taking directions, photo scales and overlaps.

This problem has been solved by means of a fully-automatic procedure that became available in March 1987. The basic concept of this program is:

*If adjustment is possible, the block geometry must be uniquely defined.
Therefore it must also be possible to compute approximate orientations
from this data, and only this data. Additional entries are not required.*

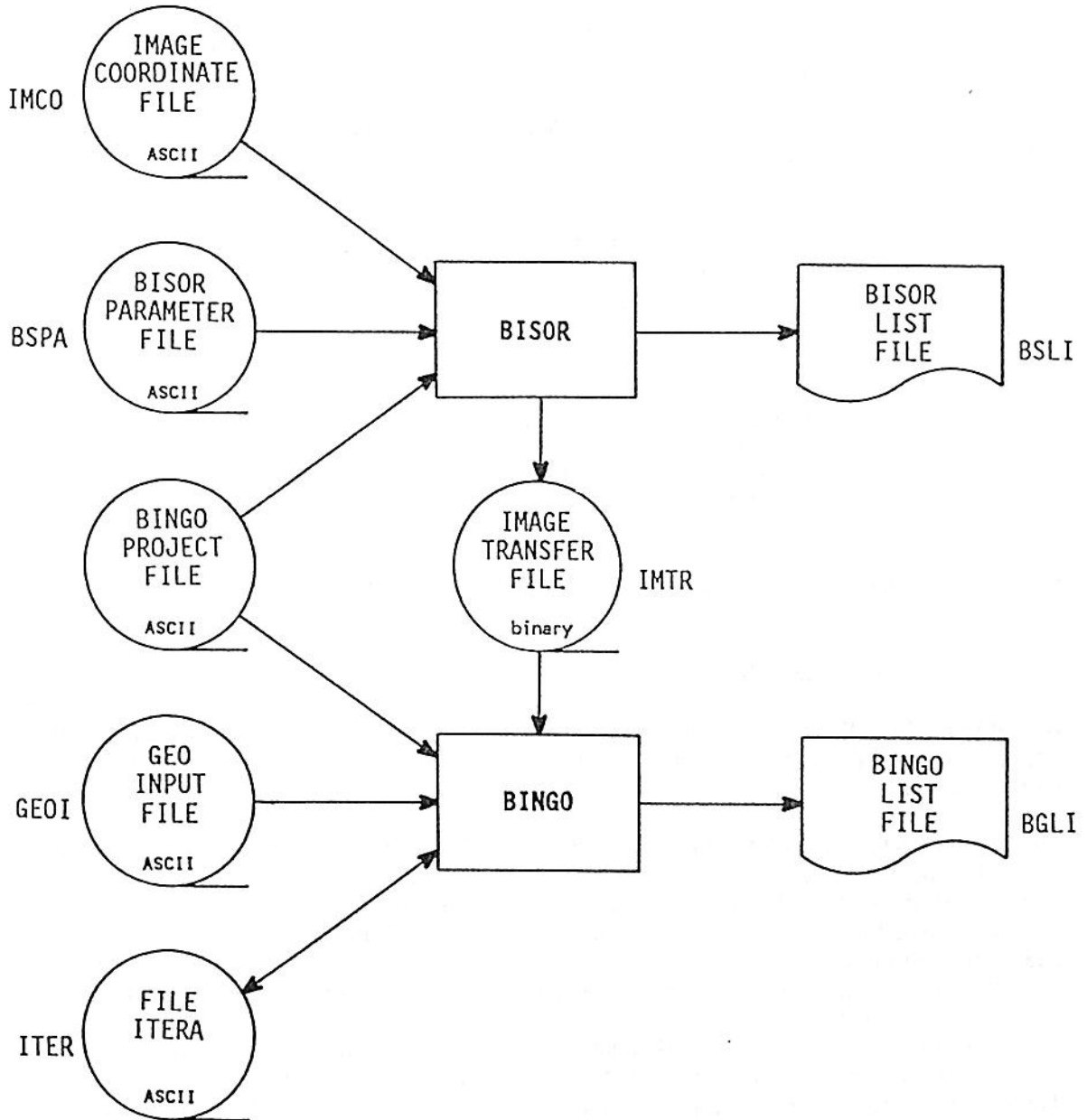
Implementing this idea was very difficult because a huge variety of different cases had to be considered. The program basically performs the following steps:

1. Determination of ties (graph), i. e. determination of the photos that have at least 6 points in common with other photos.
2. Starting out with a randomly selected photo, selection of adjacent photos until no more photo can be added to the block.
3. If possible, transformation of the block assembled in step 2 to existing control points or an adjacent block previously assembled in step 2 (combination of two blocks).
4. Search for non-oriented photos and continuation with step 2.
5. Computation of any missing point coordinates by intersection.

A big problem was relative orientation since no assumptions at all can be made on the relative location of two photos. An approach proposed by van den Hout /1976/ based on collinearity equations was expanded for practical application. Empirical tests have shown that the algorithm is very fast and rather tolerant against blunders. The formulae used will be described in another publication.

The three-dimensional Helmert transformation included in this method does not require approximations either. The solution proposed by Schut is described in Photogrammetria /1960/. The practical results obtained with this program are very positive. Computing orientation data takes markedly less time than one iteration during later bundle adjustment. This program has been called *RELAX* because the *ApproXimations* are computed by means of *RELative* orientation.

Fig. 1 BINGO Data Flow Chart



5. Data Constructs and Handling

Easy handling, protection against input errors, and user-friendly error messages are major aspects of a software package. These considerations have played a major role during the development work.

The software package is easy to handle despite the wide range of functions.

Fig. 1 shows the BINGO data flow. The package consists of two parts for historical reasons.

BISOR, a relatively small program, optimizes the image sequence to yield a favorably structured normal equation matrix, and provisionally sorts the image coordinates. Snay's Banker's algorithm /1976/ is used for optimization. The adjustment proper is computed by BINGO. A binary file is used for data transfer between BISOR and BINGO; all other files are ASCII files.

Except for the IMAGE COORDINATE FILE, which only contains the image observations, all files have the same structure:

Each line contains an input statement; it begins with a 4-character name that provides information on the statement content.

C	CAMERA PARAMETER							
C	>K<	>N<	<C'>	<XH'>	<ZH'>	<S_C'>	<S_XH'>	<S_ZH'>
CAPA	3	1	152.710	-0.020	0.010	0.050	0.020	0.020
C								
C	CONTROL POINTS							
C								
C	<NO_P>	<X>	<Y>	<Z>	<S_X>	<S_Y>	<S_Z>	
CONT	9317	34017.947	39330.952	90.311	0.005	0.005	0.001	
CONT	9556	33766.045	39878.917	6.210	0.005	0.005		
CONT	9303			119.509			0.005	

Fig. 2 Coordinates and Camera Data Entry in BINGO (Example)

All parameters have defaults to reduce data entry to a minimum for standard applications.

BINGO and BISOR are non-interactive batch programs, i. e. they enable several consecutive adjustment procedures for large blocks to be started in the evening without additional entries being required.

The IMAGE COORDINATE FILE contains refined image coordinates sorted in image sequence, and the camera number for each image.

The BISOR PARAMETER FILE contains only very little control information such as the point number range, the image coordinates format number, and similar data. In most applications this file is not required.

The PROJECT FILE specifies the disk files to be used. This file too is not required if default file names are used.

The GEO INPUT FILE, on the other hand, is very important. It contains survey data, camera data, control data etc., i. e. all the information required for adjustment.

Table 3 shows the possible input statements. After adjustment, the adjusted data is stored in the ITERA FILE for further processing or for use as approximations in another BINGO run.

The RELAX program accesses all of the files listed in the above paragraph and stores the computed orientation parameter and point coordinate approximations in the ITERA FILE for use by BINGO (Fig. 3).

Using a HP 1000 A Series computer as an example, the BINGO capabilities can be illustrated as follows: A RAM capacity of 126 K 2-byte words is enough to adjust up to 6000 unknowns. This corresponds to a block with about 400 photographs.

Table 3 BINGO Input Statements

ADPA	Additional parameters
AZIM	Azimuths
C	Comments
CANC	Cancel image coordinates for a point
CAPA	Camera parameters
COES	Empirical standard deviations for control points
CONS	Constants: earth radius and refraction coefficient
CONT	Control points
CORD	Approximate coordinates for adjustment points
DIFF	Coordinate differences in X, Y or Z
DISH	Distances horizontal
DIS3	Distances three-dimensional
ECCA	Eccentricity data for a camera (e' and δ)
ECES	Empirical standard deviations for eccentricity data
GRUP	Groups of points with a special weight
ISES	Empirical standard deviations for image station data
ISTA	Image station (camera station)
LIST	List parameters
NEWN	New number of a point (renumber)
ORIA	Exterior orientation for aerial photos
ORIT	Exterior orientation for terrestrial photos
PARM	Parameters for processing
SEDI	Sets of directions
SMCO	Stereometric camera conditions
SMES	Empirical standard deviations for stereometric camera conditions
TEXT	Text: project name
ZEDI	Zenith distances

Fig. 3 RELAX Data Flow Chart

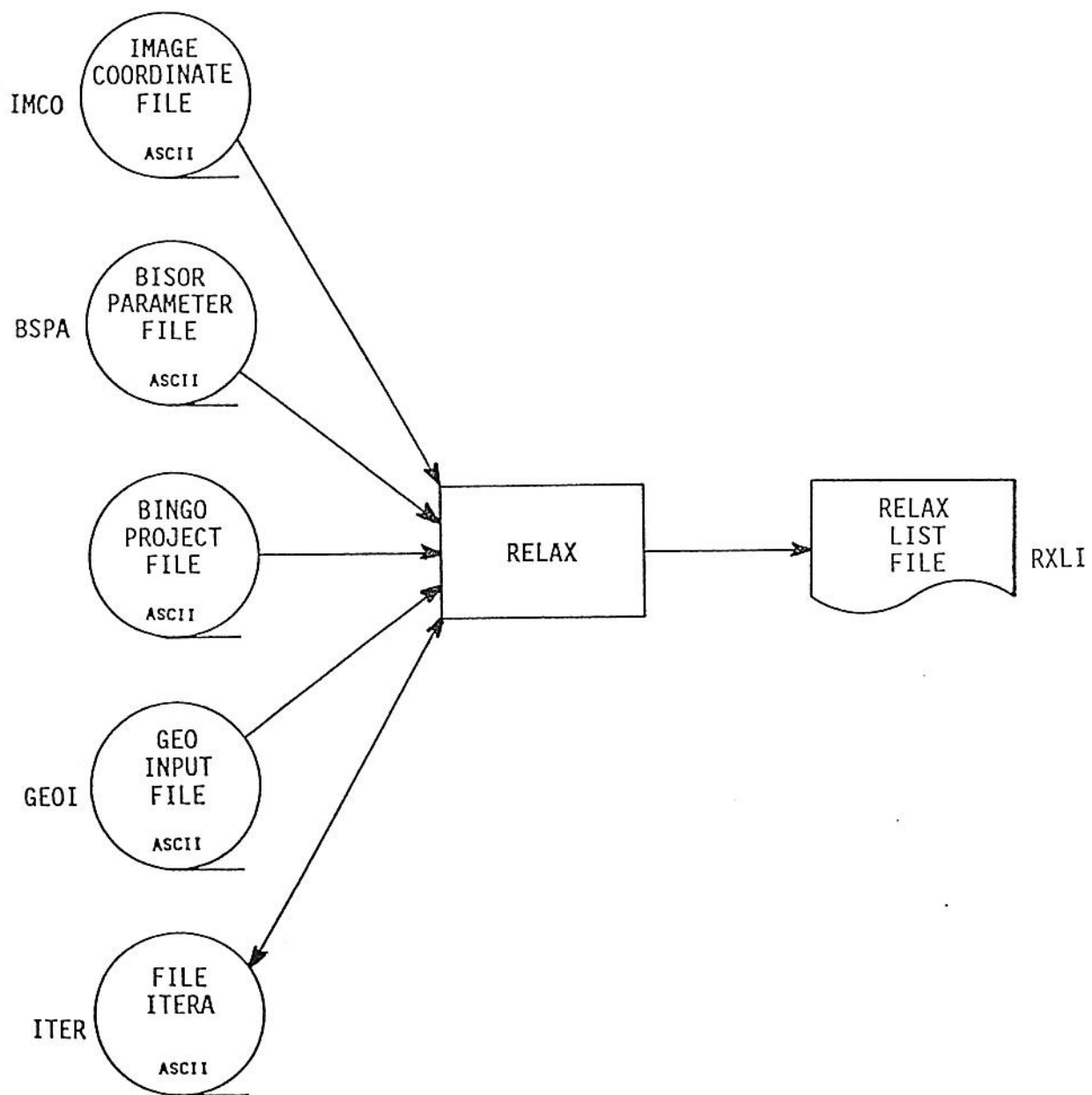
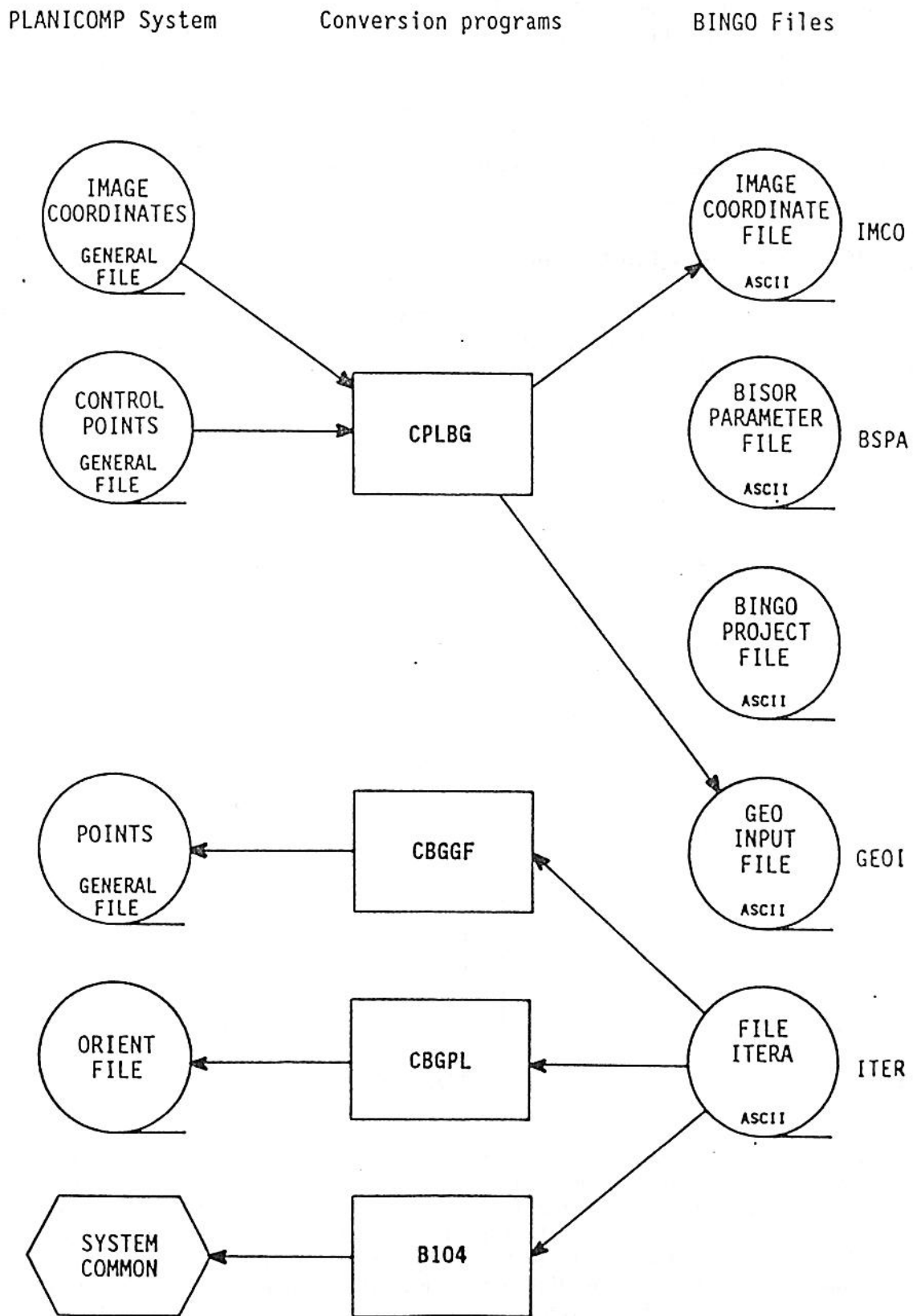


Fig. 4 Data Interchange with Planicom System



6. Linkage with the Planicom System

The Planicom is used as an example of BINGO integration in an analytical plotter environment. BINGO can run on all C- and P-Series Planicom computers.

In the C-Series environment BINGO has been integrated optimally by means of interface programs (Fig. 4). The CPLBG program (Convert from Planicom to BINGO) transfers image observations and control points from the Planicom to BINGO. B104 or CBGPL (Transfer from BINGO to Planicom) computes model parameters from the BINGO results.

So model plotting can be started immediately after the BINGO run and interior orientation without relative or absolute orientation being required. This is a very significant economical aspect. Integration in the P-Series Planicom should be completed by the end of 1987.

7. SPOT Triangulation Fundamentals

The SPOT satellite has been orbiting our earth at a height of 820 km since February 1986.

The taking system is a CCD line sensor onto which the earth surface is mapped.

The earth surface is scanned continuously by the sensor because the platform is moving (Figs. 5 and 6).

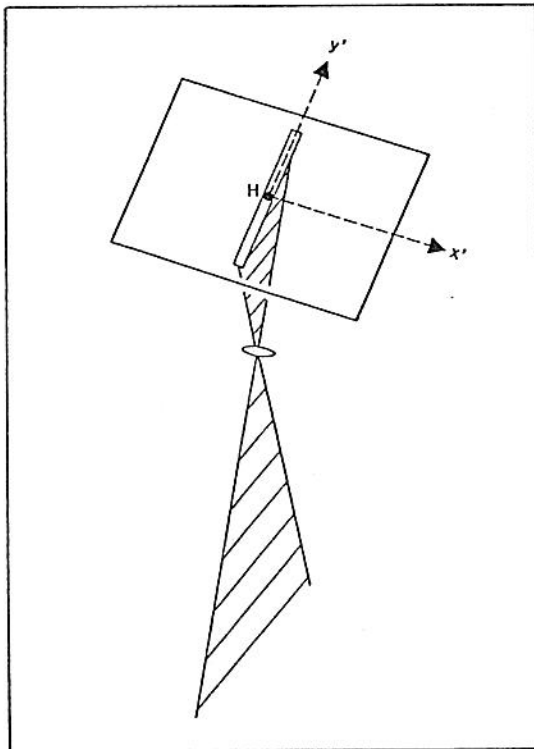


Fig. 5 Line Sensor

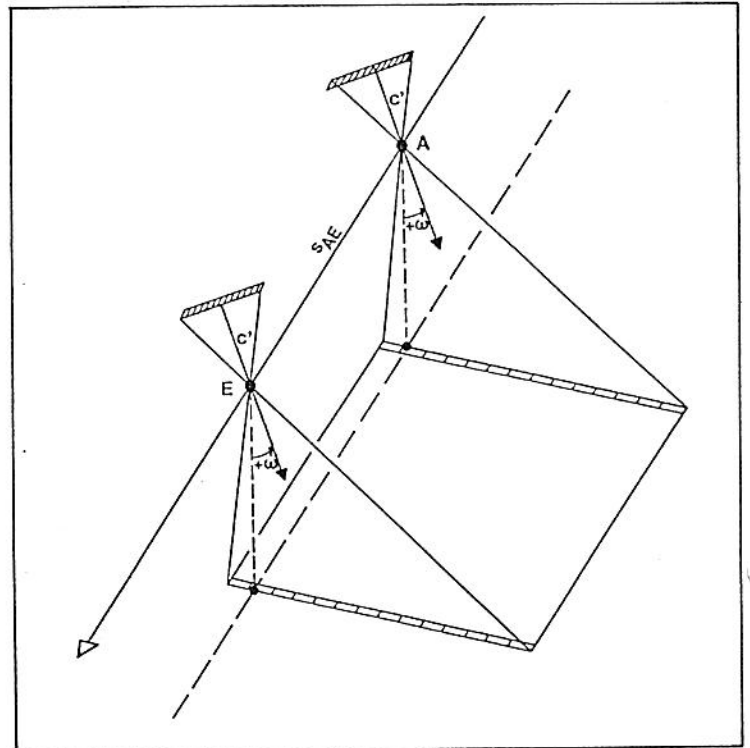


Fig. 6 SPOT Scene

All pixels of a line are read in one go at fixed time intervals of 1.5 ms (push broom scanner). The lines are therefore central-perspective, but the perspective center moves from line to line. A shear effect results from the fact that the earth rotates below the satellite while a scene is being taken. The heading h in the orbit (Fig. 7) is not the same as the direction t of the ground track. The difference r is a function of the flying speed, the flying height, the inclination, the angular speed of the earth and the geographical latitude at which the scene was taken. The orbit itself is an ellipse, but irregular accelerations cause minor deviations from the elliptical orbit. Pitch and roll (small variations of ω and φ) further distort the scene geometry. The ground distortion is very small, and corrections are possible because the irregular motion parameters are recorded. Fig. 8 shows the effect of pitch on a scene.

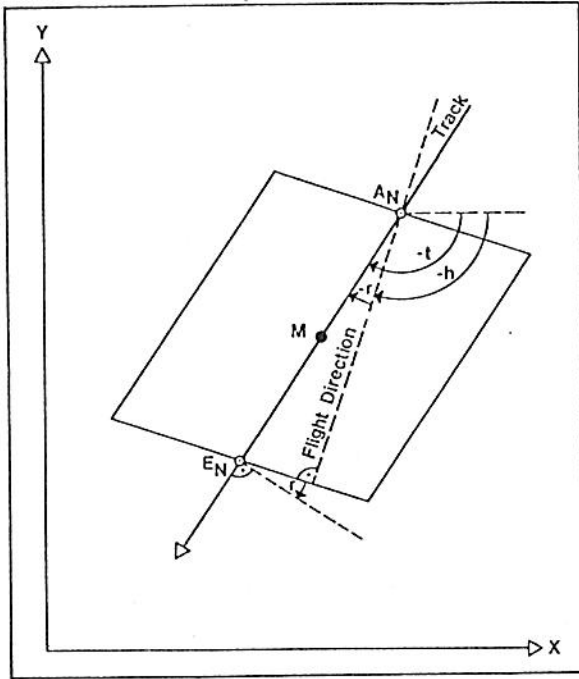


Fig. 7 Flying Direction and Track

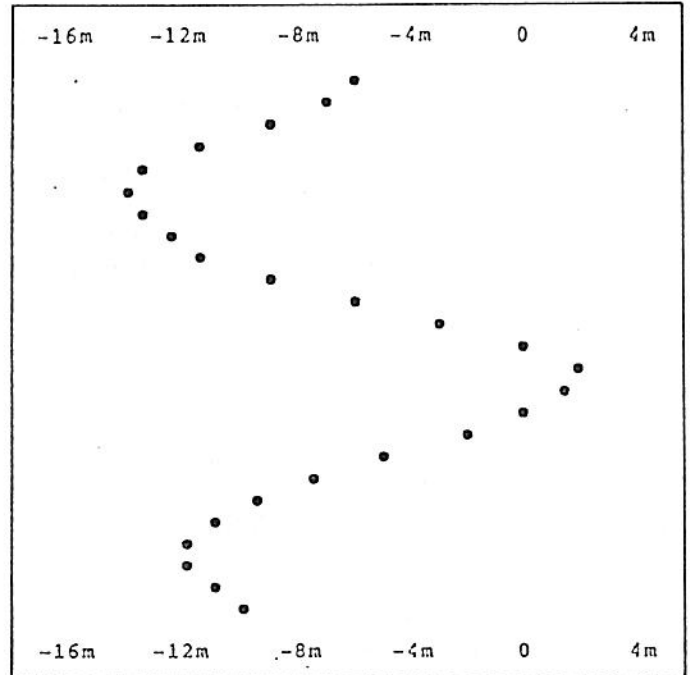


Fig. 8 Effect of Pitch

A precise mathematical model should contain a satellite motion equation and allow for the accelerations through unknown parameters that vary over time. The orbit data recordings could be used as observations /Toutin 1985/.

Another image showing the same area from another perspective would be required for later stereo restitution. Such an image pair could be bundle adjusted with the orientation parameters being time variant variables of the above-mentioned motion equation. However, such a model would be too complicated for photogrammetric practice. Therefore some simplifications are made which, for all practical intents and purposes, do not adversely affect the precision.

The orientation parameters X_0 , Y_0 , Z_0 , φ , ω and κ of a line are highly intercorrelated because the aperture angle is only 4.1 degrees. Since the ground effects are minute (< 20 m), a straight-line track can be admitted, and all irregular affects can be assumed to be variations of the angles φ , ω and κ . The φ angle variations actually are the largest irregular effect (Fig. 8).

These angle variations can easily be accounted for by additional parameters. This also applies to earth rotation correction and perspective distortions. The question which effects will most likely result from irregular motions leads to a set of 8 additional parameters (Fig. 9).

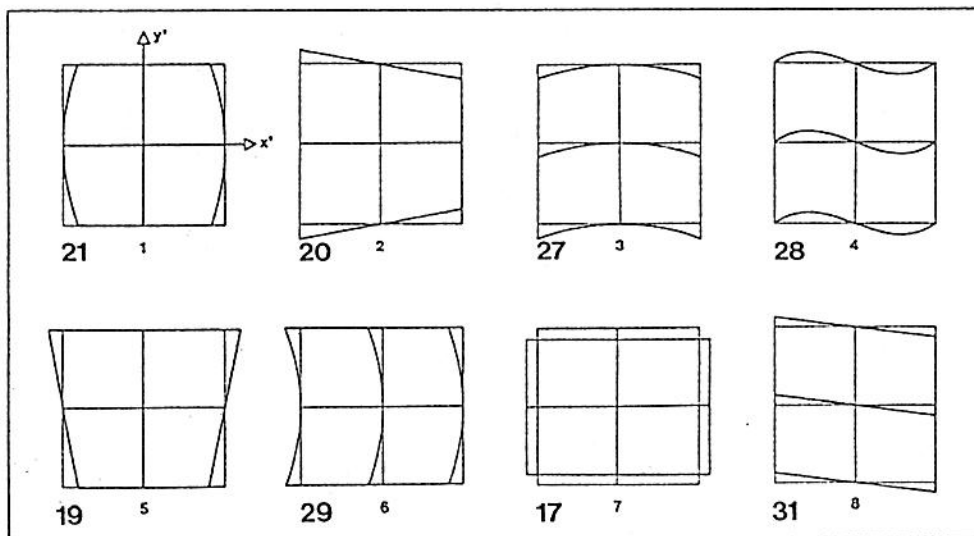


Fig. 9 Additional Parameters Describing Irregular Platform Motions

Parameter meanings:

- | | | | |
|------|----------------------------------|---|---|
| 1, 2 | Periodic variations of φ | 6 | Earth curvature correction for $\varphi \neq 0$ |
| 3, 4 | Periodic variations of ω | 7 | Affinity (overscan) |
| 5 | Periodic variations of κ | 8 | Shear (earth rotation) |

These additional parameters have been included in the BINGO bundle program. The SPOT application also required the perspective center shift to be accounted for.

$$X_{O,i} = f(\text{time } i) = f(x_i') \quad (6)$$

The image vector then is $x = (0, y', -c)$ (7)

This approach allows the angles φ , ω and κ to be considered invariable over time. If there is a sufficient number of homologous image points, the image geometry can be determined completely assuming that SPOT level 1a images that are not geometrically corrected are used.

The measured image coordinates can also be corrected beforehand for the known minor distortions of a scene (Fig. 8). Ignoring the ground elevation is permissible because the corrections are small (Fig. 8) and the resulting errors are of the order of some centimeters. SPOT IMAGE offers this prior correction on the new product level 1p. The contrast is balanced in addition to geometrical correction. At an identical pixel size of $25 \mu\text{m}$ the images are supplied in the aerial photo size of $22.5 \text{ cm} \times 22.5 \text{ cm}$. Thus one pixel corresponds to about 6.7 m on the ground compared to 10 m for level 1a.

Detail visibility has been improved in particular by local contrast enhancement.

The scale 1:266000 images allow the production of scale 1:50000 maps without any restrictions.

The product level 1p images thus correspond precisely to the mathematical concept of bundle adjustment with continuously moving perspective center and constant orientation angles φ , ω and κ described in this paper and used in BINGO.

The Planicomp analytical plotter can be used for producing line maps from SPOT models. The specific SPOT geometry has to be allowed for on-line plotting, i. e. the on-line LOOP program has to be modified because it uses the fixed-point central perspective required for conventional photogrammetric photos.

Basically there are two approaches to this problem. One solution would be to continuously vary the coordinates of the perspective center according to its current location in the image. Another approach is to describe the differences between the aerial photograph geometry and the SPOT scene geometry determined by BINGO by means of correction matrices using a mean perspective center $X_{O,P}$ (Fig.10) and taking into account that differences in elevation affect the SPOT image geometry not in the same way as the aerial photo geometry (Fig. 10). In aerial photographs, differences in terrain elevations cause a radial offset of the photo coordinates, while in SPOT images the offset is normal to the flying direction.

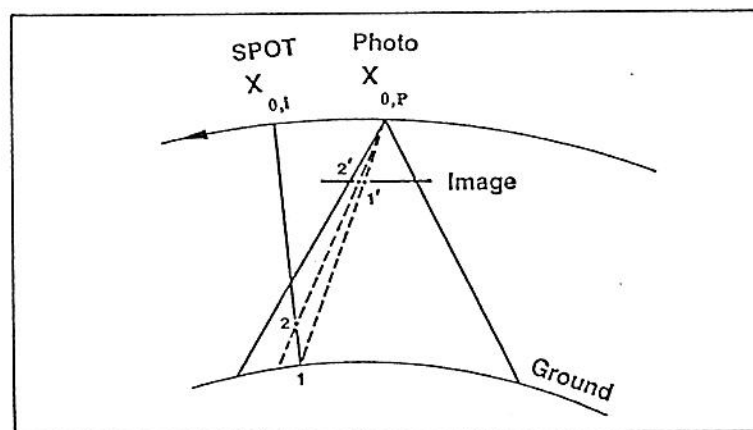


Fig. 10 Offset due to Differences in Elevation

It follows that the ground elevations should be known for establishing a correction grid. Since the elevations cannot be assumed to be known already, two correction grids are computed: one for the lowest possible ground elevation and another one for the highest possible elevation. Both are based on the image coordinates. The correction of a point P_k depends on its location in the image and its elevation Z_G in the ground system.

$$dx', dy' = f(x', y', Z_G) \quad (8)$$

The correction is computed by means of a square column (Fig. 11) /Konecny, Kruck, Lohmann 1986/ as a grid within the correction grids.

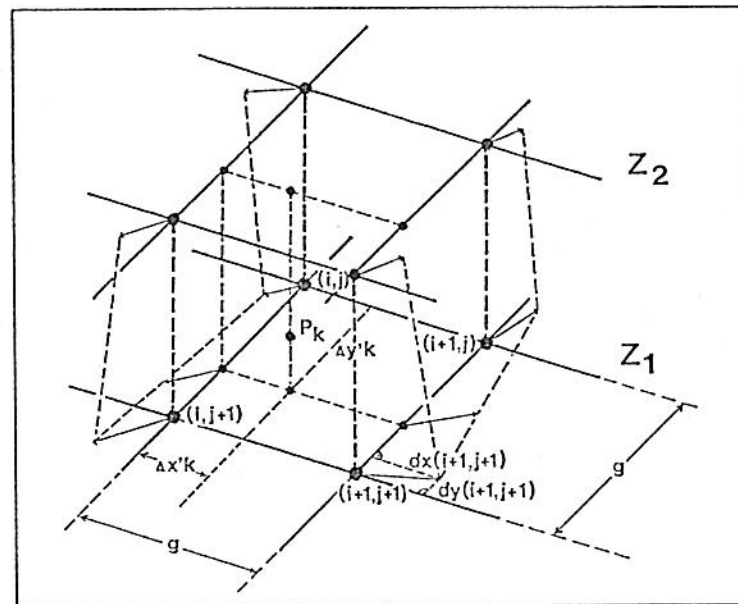


Fig. 11 Correction of the Photocoordinate during Real-Time Processing

The correction grids are established by a separate program using the following procedure:

1. Determination of the lowest ground elevation Z_{\min} .
2. Establishment of a regular grid in the image.
3. For each grid point:
 - 3.1 Computation of the associated ground coordinates X_G, Y_G using the given elevation $Z_G = Z_{\min}$, the mean perspective center $X_{0,P}$ (Fig. 10) and the collinearity equation for aerial photographs.
 - 3.4 Recomputation of the X_G, Y_G, Z_G point into the SPOT image using the shifted perspective center $X_{0,i}$ governing the representation of this point, again using the collinearity equation.
 - 3.5 Computation of the difference between the image coordinates obtained in step 3.4 and the grid point image coordinates used in step 3.1 for the initial computation.
 - 3.6 Computation of the effects of the additional parameters at the grid point, and addition of these corrections to the differences computed in 3.5. These values form the final correction for an image point x'_i, y'_i with the ground elevation Z_G .
4. Storage of the correction matrix.
5. Repetition of steps 2 to 4 for the same image but using elevation Z_{\max} instead of Z_{\min} .
6. Repetition of steps 1 to 5 for the second image.

This correction results in the SPOT stereomodel being oriented absolutely and without parallaxes in the Planicom. All existing Planicom programs can be used for plotting, and elevation and planimetry can be plotted as usual.

8. Coordinate Systems

Considering the size of a model of 60 km x 60 km, bundle adjustment must be computed in a cartesian coordinate system because the curvature of the ellipsoid has to be taken into account. Also, scale distortions of the type affecting UTM coordinates must not be allowed to interfere with adjustment because they would distort the stereo model to be plotted. This is why a local tangential coordinates system is used that should be located approximately in the center of the model.

Plotting, however, is desired in a ground coordinate system allowing for the elevations referred to the ellipsoid. Furthermore the planimetric coordinates have to be corrected in order to directly obtain coordinates in a ground coordinates system (e. g. UTM or Gauß-Krüger).

Both of these corrections can be included in the correction grids mentioned in section 7. To do this, the following two steps have to be added to the correction grid computations listed in section 7:

- 3.2 Accounting for the ellipsoid curvature, i. e. correction of the planimetric coordinates X_G , Y_G and of the elevation Z_G at this point.
- 3.3 Correction of the planimetric coordinates for the projection distortion (e. g. UTM).

This allows plotting in any desired coordinates system without increasing the computing time during on-line plotting.

Since another coordinates system and another ellipsoid are generally used for plotting than for satellite orbit data acquisition, it is deemed advisable to always use at least four control points. Only then can a satisfactory ellipsoid transition be ensured.

9. Conclusions

The brief description of BINGO and the SPOT approach given in this paper are only a rough outline that is going to be filled in and expanded by the following lectures. The implemented orthophoto solution for the Orthocomp has not even been mentioned yet, nor have digital orthophotos. These subjects are left open for further lectures.

The following can be said regarding the long-term enhancement of BINGO: Considerations are under way on how to include GPS data in adjustment - both for ground use and for position determination in the plane. Relative observations in particular should be considered for plane use. Data of this type would also be interesting for the SPOT application. However, it still remains to be seen whether GPS will become an operational system that will be available all day long.

On the practical use of the SPOT data I should like to say in conclusion that this trend-setting technology has attained an operational status but is being introduced in photogrammetric practice only sluggishly. This is certainly not due to the plotting methods, the availability or the precision. The level 1p image quality is excellent. If lacking information should be the reason, we hope that this seminar will promote increased SPOT data usage.

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